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Turbulent Wake Characteristics with Different Eddy Viscosity Coefficients

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In the studies of turbulent hypersonic wakes, the viscosity coefficient for laminar wake is generally replaced by an eddy viscosity coefficient The forms of the eddy viscosity coefficient assumed in various theories[1–5] may be classified into three general types They are

$$\mu_{\epsilon_1} = K_1\delta(\rho_e u - \rho\, u\,) \tag{1}$$

$$\mu_{\epsilon_2} = K_2\delta\rho_e(u - u\,) \tag{2}$$

$$\mu_{\epsilon_3} = K_3\delta\rho\,(u - u\,) \tag{3}$$

where the K's are constants to be assumed All these three forms of the eddy viscosity coefficients reduce to the same expression generally used in the incompressible turbulent free-mixing problem, i e , $K_i\rho_i\delta_i(u_{\,i} - u_{\,i})$

This note gives and compares some numerical results of the equilibrium turbulent-wake characteristics obtained by using these three different forms of the eddy viscosity coefficients with the governing equations, method of calculations, and initial and boundary conditions being the same for all three cases

To simplify the analysis, the usual wake equations of the boundary-layer type approximation are further reduced by assuming the Lewis number and the Prandtl number equal to one, and the axial pressure gradient as negligible The governing equations are, then, in axisymmetric coordinates

$$\frac{\partial(\rho u)}{\partial x} + \frac{1}{r}\frac{\partial(\rho v r)}{\partial r} = 0 \tag{4}$$

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_\epsilon\frac{\partial u}{\partial r}\right) \tag{5}$$

$$\rho u\frac{\partial h}{\partial x} + \rho v\frac{\partial h}{\partial r} = \mu_\epsilon\left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_\epsilon\frac{\partial h}{\partial r}\right) \tag{5a}$$

It is well known that the following relation holds in this case, i e ,

$$h + (u^2/2) = A + Bu \tag{6}$$

where A and B are constants If the initial condition also satisfies (6), then this relation between the enthalpy and the velocity is valid for the whole viscous flow regime It remains now to solve Eq (5)

To do this, the integral method is employed The integration of Eq (5) from $r = 0$ to $r = \delta$ gives

$$\frac{d}{dx}\left[\rho\, u\,^2\delta^2\int_0^1 \frac{\rho u}{\rho\, u}\left(\frac{u}{u} - 1\right)\frac{r}{\delta}\,d\left(\frac{r}{\delta}\right)\right] = 0 \tag{7}$$

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where Eq (4) and assumption $du\,/dx \cong 0$ have been used, and the boundary conditions $\partial u/\partial r = 0$ at $r = 0$ and $r = \delta$ have been enforced Equation (7) can then be rewritten as

$$\rho\, u\,^2\delta^2\theta = \text{const} \tag{7a}$$

where

$$\theta = \int_0^1 \frac{\rho u}{\rho\, u}\left(\frac{u}{u} - 1\right)\frac{r}{\delta}\,d\left(\frac{r}{\delta}\right) = \int_0^1 \frac{\rho u}{\rho\, u}\left(\frac{u}{u} - 1\right)\eta d\eta$$

The momentum balance along the axis gives another condition as

$$\left[\rho u\left(\frac{\partial u}{\partial x}\right)\right]_{=0} = \left(2\mu_\epsilon\frac{\partial^2 u}{\partial r^2}\right)_{=0}$$

It can be shown that this is equivalent (see Ref 6), in nondimensional form, to

$$\left(\bar\rho\bar u\,\frac{\partial\bar\rho\bar u}{\partial\bar x}\right)_{\eta=0} = \left(\frac{\mu}{\rho\, u\,\delta_0}\,\frac{2\bar\mu_\epsilon}{\bar\delta^2}\,\frac{\partial^2\bar\rho\bar u}{\partial\eta^2}\right)_{\eta=0} \tag{8}$$

where $\bar\rho = \rho/\rho$, $\bar u = u/u$, $\bar\mu_\epsilon = \mu_\epsilon/\mu_e$, $\bar x = x/\delta_0$, $\bar\delta = \delta/\delta_0$ and δ_0 is the initial viscous wake radius

Now, the following profile for $\bar\rho\bar u$ is assumed,

$$\bar\rho\bar u = F(\eta) + a[1 - F(\eta)] \tag{9}$$

where $F(\eta)$ is the given initial profile for $\bar\rho\bar u$ at $x = 0$ Substituting (9) into (8) results in

$$\frac{da}{d\bar x} = \frac{\mu_e}{\rho\, u\,\delta_0}\,\frac{2\bar\mu_\epsilon}{\bar\delta^2}\,\frac{F_0''(1 - a)}{(1 - F_0)[F_0 + a(1 - F_0)]} \tag{10}$$

where $F_0 = F(0)$ and $F_0'' = (\partial^2F/\partial\eta^2)_{\eta=0}$ The solution of Eqs (10) and (7a) yields $\bar\delta$ and a as functions of $\bar x$ and hence the distribution of $\bar\rho\bar u$ With the thermodynamic relation $\rho = f_1(h,p)$ or $h = f_2(p,\rho)$ and relation (6), $\bar h$ and $\bar u$ can be calculated The wake characteristics in the whole field can then be obtained

To study the effect of the different eddy viscosity coefficients given in (1–3) on the wake characteristics, it remains to substitute these coefficients into (10) in turn, and obtain the solution for the individual case using the same initial and boundary conditions

This has been applied to the case of a slender cone at altitude = 150,000 ft and M_∞ = 19 52 The external inviscid flow conditions were assumed uniform The initial flow profiles used in the calculations are shown in Fig 1 It is noted that the peak enthalpy is located off the axis for the initial profiles so assumed The results of the variations of the centerline enthalpy $\bar h$ with $2K\bar x$ for the three cases are shown in Fig 2, and those of the centerline velocity $\bar u_c$ and the eddy viscosity coefficient $\bar\mu_\epsilon'$ in Fig 3

From Fig 2, it is seen that the distance which the off-axis peak enthalpy takes to reach the axis differs significantly

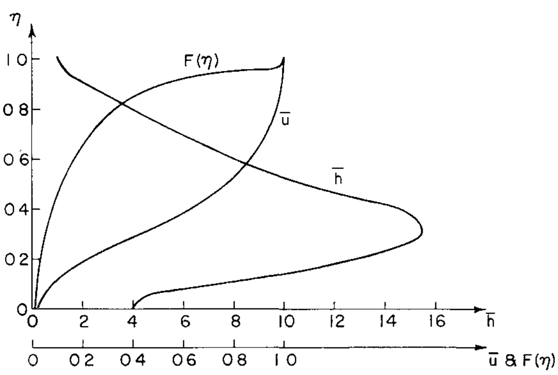

Fig 1 The initial profiles; slender cone, altitude = 150,000 ft, M_∞ = 19 52

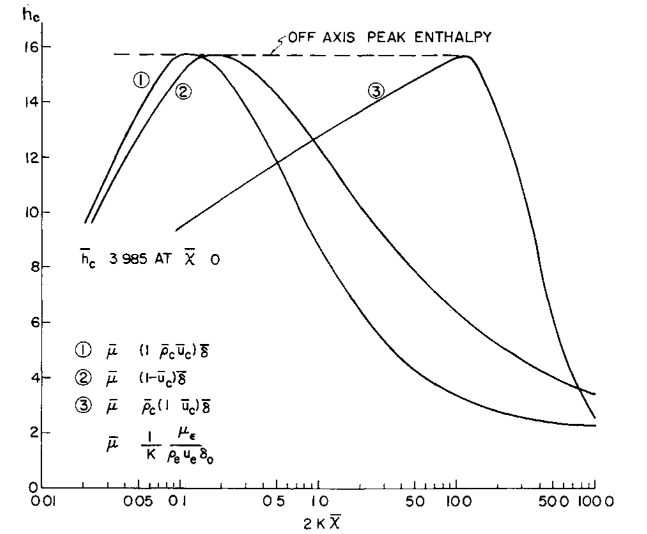

Fig 2 Variations of axis enthalpy; slender cone, altitude = 150,000 ft, M_∞ = 19 52

for the three different forms of the eddy viscosity coefficients For case 3, this distance is $(2K\bar{x})_{\bar{h}_{cm\,x}} = 10$, which is two orders of magnitude higher than case 1, where $(2K\bar{x})_{\bar{h}_{cm\,x}} = 0\ 1$, whereas in case 2, $(2K\bar{x})_{\bar{h}_{cmax}} = 0\ 2$ The constant K is generally considered to be in the order of 10^{-2} whereas the initial viscous wake width is in the order of the base radius of the body Therefore, in terms of the physical distance, the off-axis peak enthalpy reaches the axis at a distance approximately equal to 500 base radii from the initial axial station for case 3, whereas it is only 10 and 5 base radii for cases 2 and 1, respectively Since the peak enthalpy is off the axis from $\bar{x} = 0$ up to $\bar{x} = \bar{x}_{\bar{h}_{cm\,x}}$, hence the hottest portion of the wake also spreads through this region This hottest portion of the wake is, therefore, approximately 500 base radii long for case 3, compared with 10 and 5 base radii for cases 2 and 1, respectively, for the conditions used in the present calculations Since the external inviscid conditions are assumed uniform, the electron density distribution follows the same trend as the enthalpy distribution

It is further noted that the variations of the centerline enthalpy for these three cases can be predicted qualitatively by the values of their corresponding eddy viscosity coefficients (Fig 3) In the present case, the centerline enthalpy is larger than the inviscid enthalpy Therefore, ρ is smaller than ρ and hence the value of μ_{ϵ_3} is smaller than μ_{ϵ_2} and they are in turn smaller than $\mu_{\ 1}$ [Eqs (1–3)] With a smaller μ_ϵ or slower diffusion process, a slower axial change of a [Eq (10)] and hence those of $\bar{\rho}\bar{u}$, $\bar{u}$, and $\bar{h}$ result This is clearly indicated in Figs 2 and 3

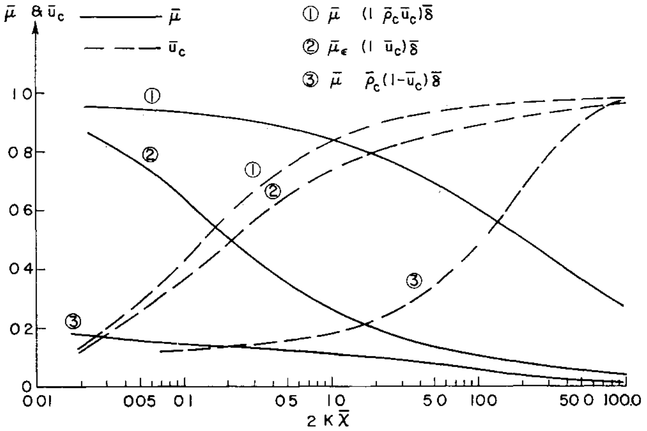

Fig 3 Variations of axis velocity and eddy viscosity coefficient; slender cone, altitude = 150,000 ft, M_∞ = 19 52

The forementioned reasoning generally applies to a slender cone under hypersonic flight conditions Therefore, the significant differences in the equilibrium turbulent-wake characteristics due to the different forms of the eddy viscosity coefficient as given here may be considered as typical for a hypersonic slender cone

Since all the three forms of the eddy viscosity coefficients considered here reduce to the same form in incompressible case but give different results for the wake characteristics, firmer ground derived from experimental evidence on which the selection of the form of the eddy viscosity coefficient can be based is much needed †

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† After this work had been completed, Ref 7 came to the author's attention The same problem treated here is also considered in that work with a finite difference method of solution, using the exact expressions given in Refs 2–4 and 8 The conclusions agree qualitatively with the present work

Correlation of Boost Phase Turbulent Heating Flight Data

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Nomenclature

c_p = specific heat at constant pressure
G = axisymmetric factor defined by Eq (3)
h = convective heat transfer coefficient
k = coefficient of thermal conductivity
Nu^* = Nusselt number, hs/k^*
P = pressure
Pr^* = Prandtl number, $\mu^* c_p^*/k^*$
r = radius of surface point from axis of symmetry
R_b = nose radius of curvature
Re^* = Reynolds number, $\rho^* u\, s/\mu^*$
s = boundary-layer length
u = velocity component parallel to surface
γ = specific heat ratio
μ = dynamic viscosity
ρ = density

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